Giant Gravitons in Twisted Holography w/ Davide Gaiotto

Twisted holography setup [Costello, Gaiotto]

B-model topo string on deformed conifold $SL(2,\mathbb{C})$* and coupling N^{-1} $\Longleftrightarrow$ large N expansion of chiral algebra $\mathcal{A}_N$ = gauged $\beta\gamma$ system in adj. $\mathfrak{u}(N)$

* with appropriate boundary conditions

"derivation"

B-model on $\mathbb{C}^3$ + N D1 branes $\xrightarrow{\text{backreacts}}$ B-model on $SL(2,\mathbb{C})$

$\mathcal{A}_N$ ↗ (N D1 branes)

This is also the protected subsector of AdS_5 / CFT_4 :

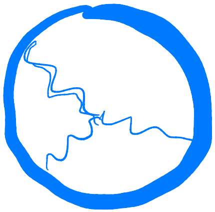
$\mathcal{N}=4$ SYM with $U(N)$ $\longleftrightarrow$ type IIB on $AdS_5 \times S^5$

$\downarrow$ Beem et al. twist / holomorphic topological twist on Ω

$\downarrow$ twisted strings / SUGRA [Costello, Li, ...]

chiral algebra $\mathcal{A}_N$ — B-model on $SL(2,\mathbb{C}) \approx AdS_3 \times S^3$

Holographic dictionary:



$\mathcal{O}(1)$ i.e. finite size in $N \to \infty$ | local modifications of asymptotic boundary condition

single traces

They matched 2pt and 3pt correlation functions by matching the global symmetry algebra

We will extend this to include:

$\mathcal{O}(N)$: determinants, subdeterminants | Giant Graviton D2 branes

Work in progress:

$\mathcal{O}(N^2)$: $(\det X)^N$ | backreacted geometries

Plan:

* $\mathcal{O}(1)$, $\mathcal{O}(N)$ operators in chiral algebra $\mathcal{W}_N$
* correlation functions of determinants
* define a spectral curve for each saddle of ↑
* holographic checks
* future directions

Chiral algebra $\mathcal{A}_N$

by Beem et al. twist of a gauge theory is a gauged $\beta\gamma$ system

in our case:

symplectic bosons X, Y in adj. of $u(N)$: $X^a_b(z) Y^c_d(w) \sim \delta^a_d \delta^c_b \frac{1}{N} \frac{1}{z-w}$

bc system in $u(N)$: $b_I(z) c^J(w) \sim \delta^J_I \frac{1}{N} \frac{1}{z-w}$

$Q_{BRST} \sim N \oint \mathrm{Tr}\left(c[X,Y] + \frac{1}{2} b[c,c] \right)$

$\mathcal{O}(1)$ operators (finite size in $N \to \infty$)

basic operators are single-traces

in large N the BRST cohomology of single-traces generated by:

$$A^{(n)} = \mathrm{Tr}\, \mathcal{Z}^{(i_1} \mathcal{Z}^{i_2} \cdots \mathcal{Z}^{i_n)} \quad \Leftarrow (\mathcal{Z}^1, \mathcal{Z}^2) := (X, Y)$$

$$B^{(n)} = \mathrm{Tr}\, b\, \mathcal{Z}^{(i_1} \mathcal{Z}^{i_2} \cdots \mathcal{Z}^{i_n)}$$

$$C^{(n)} = \mathrm{Tr}\, \partial c\, \mathcal{Z}^{(i_1} \mathcal{Z}^{i_2} \cdots \mathcal{Z}^{i_n)}$$

$$D^{(n)} = \frac{1}{2} \varepsilon_{ij}\, \mathrm{Tr}\, \partial \mathcal{Z}\, \mathcal{Z}^{(j} \mathcal{Z}^{i_2} \cdots \mathcal{Z}^{i_n)}$$

$\mathrm{Tr}\, XXYYY + \mathrm{Tr}\, XYYXY$

$\mathcal{O}(1)$ single-traces are dual to modifications of boundary conditions in B-model on $SL(2,\mathbb{C})$

to talk about asymptotic boundary conditions we compactify

$$SL(2,\mathbb{C}) = \{ ad - bc = 1 \} \subset \mathbb{C}^4$$

to

$$\overline{SL(2,\mathbb{C})} = \{ AD - BC = V^2 \} \subset \mathbb{CP}^4$$

boundary divisor is $D = \{ AD - BC = 0 \} \subset \mathbb{CP}^3$

$$\approx \mathbb{CP}^1_{C/A} \times \mathbb{CP}^1_{B/A}$$

Euclidean

topologically $SL(2,\mathbb{C}) \approx AdS_3 \times S^3$

$$\downarrow S^1$$

$$\partial AdS_3 \approx \mathbb{CP}^1_{C/A} \times \mathbb{CP}^1_{B/A}$$

O(N) operators: determinants and subdeterminants

they are BRST invariant

$$\det \mathcal{Z} = \frac{1}{N!} \varepsilon_{i_1 \dots i_N} \varepsilon^{j_1 \dots j_N} \mathcal{Z}^{i_1}_{j_1} \mathcal{Z}^{i_2}_{j_2} \dots \mathcal{Z}^{i_N}_{j_N} = \frac{1}{N!} \varepsilon\varepsilon(\mathcal{Z}, \dots, \mathcal{Z})$$

$$\det_\ell \mathcal{Z} = \frac{1}{N!} \binom{N}{\ell} \varepsilon_{i_1 \dots i_\ell i_{\ell+1} \dots i_N} \varepsilon^{j_1 \dots j_\ell i_{\ell+1} \dots i_N} \mathcal{Z}^{i_1}_{j_1} \dots \mathcal{Z}^{i_\ell}_{j_\ell}$$

$$=: \frac{1}{N!} \binom{N}{\ell} \varepsilon\varepsilon(\underbrace{\mathcal{Z}, \mathcal{Z}, \dots}_{\ell}, \underbrace{\mathbb{1}, \mathbb{1}, \dots}_{N-\ell})$$

generating function for subdets:

$$\det(M + \mathcal{Z}) = \sum_{\ell=0}^{N} M^{N-\ell} \det_\ell \mathcal{Z}$$

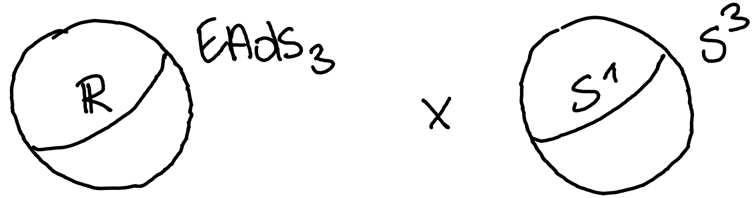
we define:

$$\mathcal{Z}(u, z) := X(z) + u X(z)$$

$$D(M, u, z) := \det(M + \mathcal{Z}(u, z))$$

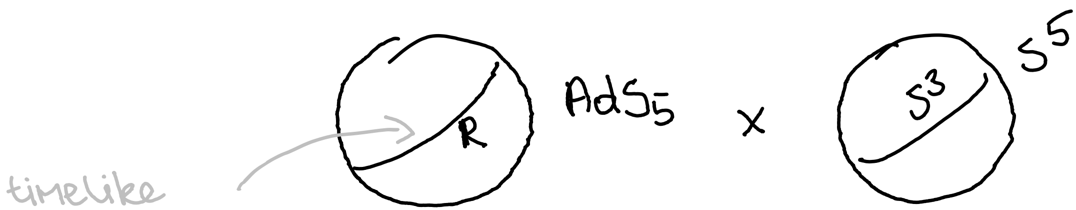
Dets and subdets are dual to Giant Gravitons in B-model

GGs are D2 branes that wrap $\mathbb{C}^*$:



near boundary

In $AdS_5 \times S^5$ GGs are D3 branes :

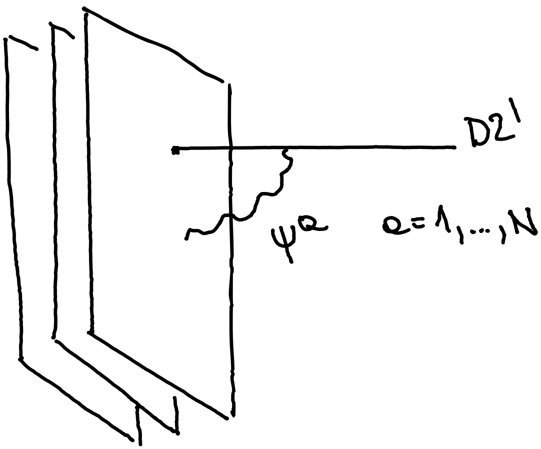


near boundary

physical strings : D3 brane : 3 (real) spatial + 1 time dim

topo strings : D2 brane : 2 (real) spatial dim

How is this duality "derived" ?



ND2

open strings between ND2 – D2' couple to the worldvolume action of ND2 as

$$\int e^{\bar{\psi}_a \chi^a{}_b \psi^b}$$

after we integrate out fermions we get a determinant

$$\det \chi$$

Backreaction without extra probe D2' branes:

$(a,b,z) \in \mathbb{C}^3$ + N D2 branes at $a=b=0$

$N \int_{D2} \partial^{-1} \alpha$

eom in the presence of branes:

$\Omega^{2,2} \approx PV^{1,2}$

$$\bar{\partial}\alpha + \frac{1}{2}\{\alpha,\alpha\} + N\,\delta_{a=b=0} = 0 \qquad (*)$$

Beltrami differential β that solves (*) defines a new complex structure on $\mathbb{C}^3 \setminus \mathbb{C}$:

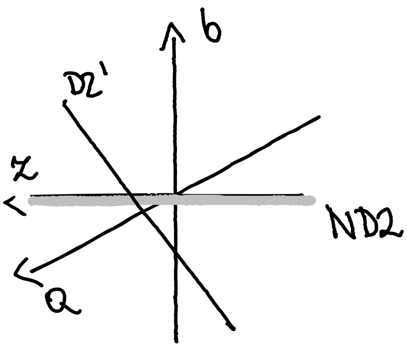
a, b stay holomorphic coords

(az, bz) get deformed $\quad c = az + \ldots, \quad d = bz + \ldots$

New holomorphic coords satisfy:

$$ad - bc = 1$$

Backreaction with extra probe D2' brane:



N D2 wrap $a = b = 0$

D2' wraps $m_0 + b - u_0 a = 0\,, \quad z = z_0$

boundary behaviour of backreacted D2' :

$$\frac{b}{a} = u_0 - \frac{m_0}{a} + \dots \qquad \mathbb{CP}^1_{B/A}$$

$$\frac{c}{a} = z_0 + \dots \qquad \mathbb{CP}^1_{C/A} \qquad \partial AdS_3$$

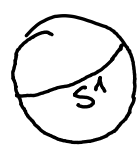
$\det(M + Z(u,z))$ is dual to GG brane with boundary conditions:

$$\frac{b}{a} = u - \frac{M}{a} + \dots$$

$$\frac{c}{a} = z + \dots$$

$\det(M + Z(u,z))$

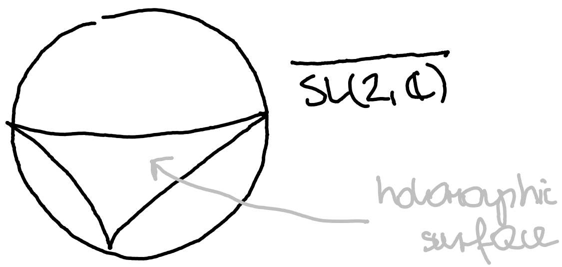
- (u,z) ← position on ∂AdS_3
- Z ← controls orientation of $S^1 \subset S^3$
- M ← controls size of $S^1 \subset S^3$

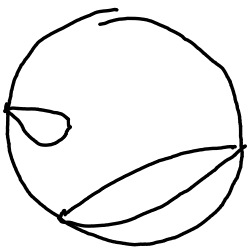
e.g. $M = 0$ maximal 

S^3

More determinants? $\det(m_i + Z(u_i, x_i))$

many holomorphic surfaces in $SL(2,\mathbb{C})$ that satisfy boundary conditions





we will match saddles $\mathfrak{s}$ of correlation functions of determinants with brane configurations

m_i, u_i, x_i control boundary behaviour

"$\mathfrak{s}$" will control the shape in the bulk

How to compute correlation functions of dets [Jiang, Komatsu, Vescovi]

- fermionize dets $\quad \det(M + Z(u,x)) = \int [d\psi d\bar\psi] \, e^{\bar\psi (M + Z(u,x))\psi}$

$$\bar\psi_a (M \delta^a_b + Z^a_b)\psi^b$$

$$\left\langle \prod_{i=1}^{k} D(M_i, u_i, x_i) \right\rangle = \int \prod_i [d\psi_i d\bar\psi_i] \left\langle \prod_i e^{\bar\psi_i (M_i + Z(u_i, x_i))\psi_i} \right\rangle$$

- integrate out bosons

$$= \int \prod_i [d\psi_i d\bar\psi_i] \, e^{-\frac{1}{2N} \sum_{i \neq j} \frac{u_i - u_j}{x_i - x_j} (\bar\psi_i \psi_j)(\bar\psi_j \psi_i) + \sum_i M_i \bar\psi_i \psi_i}$$

- introduce auxilliary bosonic variables ρ^i_j $i \neq j$ $\quad$ (and $\rho^i_i := M_i$)
(Hubbard - Stratonovich transformation)

$$= \frac{1}{Z_\rho} \int \prod_i [d\psi_i d\bar\psi_i] \int \prod_{i \neq j} [d\rho^i_j] \, e^{\frac{N}{2} \sum_{i \neq j} \frac{x_i - x_j}{u_i - u_j} \rho^i_j \rho^j_i + \sum_{i,j} \rho^i_j \bar\psi_i \psi_j}$$

$$\Downarrow$$

$$(\det \rho)^N$$

- integrate out fermions:

$$\langle \prod_i D_i \rangle = \frac{1}{Z_\rho} \int d\rho \, e^{N S[\rho]}, \quad S[\rho] = \frac{1}{2} \sum_{i \neq j} \frac{z_i - z_j}{u_i - u_j} \rho^i_j \rho^j_i + \log\det\rho$$

- in large N we can do saddle pt approximation

saddle eqs $\qquad \frac{z_i - z_j}{u_i - u_j} \rho^i_j + [\rho^{-1}]^i_j = 0, \quad i \neq j$

in the matrix form

$$[\zeta, \rho] + [\mu, \rho^{-1}] = 0$$

we are solving for off-diag.

where

$$\zeta = \begin{pmatrix} z_1 & & & \\ & z_2 & & \\ & & \ddots & \\ & & & z_k \end{pmatrix}, \quad \mu = \begin{pmatrix} u_1 & & & \\ & u_2 & & \\ & & \ddots & \\ & & & u_k \end{pmatrix}, \quad \rho^i_i = m_i$$

- for later, define p_i conjugate to m_i : $\quad p_i = \frac{\partial S}{\partial m_i} = [\rho^{-1}]^i_i$

Conjecture

Saddles g^* that solve $[\zeta, g] + [\mu, g^{-1}] = 0$

correspond to Giant Graviton branes in B-model on $SL(2,\mathbb{C})$.

For each g^* we will define a spectral curve S_{g^*} in $SL(2,\mathbb{C})$

and check it matches GG brane.

Comparison to $AdS_5 \times S^5$:

- t'Hooft coupling λ ?
- $\langle \det \det \rangle$, $\langle \det \det \det \rangle$ for $\frac{1}{2}$BPS are tree level exact [Jiang, Komatsu, Vescovi]
- we'd have to find SUGRA solutions corresponding to GGs

Spectral curve

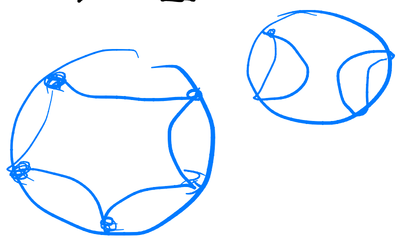
For any g^* (that solves saddle eqs) define commuting matrices:

$$B(a) = a\mu - g, \quad C(a) = az + g^{-1}, \quad D(a) = az\mu + g^{-1}\mu - zg$$

which satisfy

$$aD(a) - B(a)C(a) = 1 \qquad \forall a$$

Define spectral curve S_{g^*}:



$\{ (a,b,c,d)$ s.t. b,c,d are simultaneous eigenvalues of $B(a), C(a), D(a) \}$

$k \times k$

S_{g^*} comes with line bundle $\mathcal{L}_{g^*}$: common eigenline of $B(a), C(a), D(a)$

Spectral curve properties

Boundary behaviour

$a \to \infty$:

$$\frac{B(a)}{a} = \mu - \frac{S}{a} = \begin{pmatrix} u_1 - \frac{M_1}{a} & & \\ & \ddots & \\ & & u_k - \frac{M_k}{a} \end{pmatrix} + \ldots$$

$$\frac{C(a)}{a} = z + \frac{S^{-1}}{a} = \begin{pmatrix} x_1 + \frac{p_1}{a} & & \\ & \ddots & \\ & & x_k + \frac{p_1}{a} \end{pmatrix} + \ldots$$

so as $a \to \infty$ there are k branches (a, b_i, c_i, d_i)

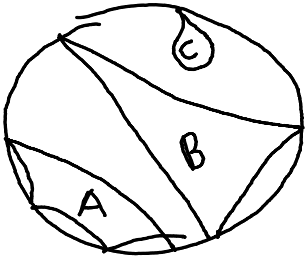
$$\frac{b_i}{a} = u_i - \frac{M_i}{a} + \ldots \quad , \quad \frac{c_i}{a} = x_i + \frac{p_i}{a} + \ldots$$

Matches the asymptotic boundary conditions of Giant Graviton brane corresponding to k determinants $D(M_i, u_i, x_i)$.

Irreducible and reducible saddles

we can consider block diagonal saddles:

$$S = \begin{bmatrix} S_A & & \\ & S_B & \\ & & S_C \end{bmatrix}$$

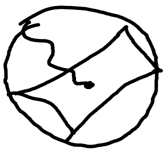


disconnected spectral curve

Various holographic checks

* $S[g^*]$ vs $S[\text{brane}]$
* correlation functions of dets with a trace

$$\langle \det \det \ldots \operatorname{tr} \rangle \sim$$



* modifications of determinant / excitations of brane

Compare actions

chiral algebra side: $S[g] = \frac{1}{2} \sum \frac{z_i - z_j}{u_i - u_j} g^i_j g^j_i + \log \det g$

conjugate coords: $x_i \, , \; p_i = \frac{\partial S}{\partial x_i} = [g^{-1}]^i_i$

B-model side: worldvolume theory of D2 brane is a $\beta\gamma$ system

$$\int_{S_{g^*}} \beta \bar{\partial} \gamma \wedge \frac{da}{a} \qquad (*)$$

fluctuations in 2 normal directions to the brane: b, c

close to the brane expand β, γ in powers of a:

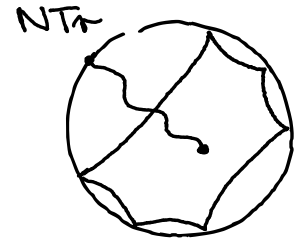
$$\beta(a) = \sum \beta_n a^n \quad , \quad \gamma(a) = \sum \gamma_n a^n$$

close to the boundary: $b = a u_i - x_i + \dots \, , \quad c = a z_i + p_i + \dots$

so the zero modes are $\beta_0 = -x_i \, , \; \gamma_0 = p_i$ and they are conjugate wrt action $(*)$

Correlation functions of determinants with a single trace

$\langle \det \det \ldots N\mathrm{Tr} \rangle \big|_{N\to\infty}$ dual to



tree Witten diagram

large N also controlled by the same saddles ρ^*

$$\left\langle \prod_i D_i \; N\mathrm{Tr}\, z^n \right\rangle \Big|_{N\to\infty} = \frac{1}{Z_\rho} \int d\rho \; e^{N S[\rho]} \Big(-N\mathrm{Tr}_{k\times k} \big(\underbrace{-\rho \frac{\mu-u}{z-x}}_{R(u,z)} \big) \Big)$$

in the saddle pt approx: $- e^{NS[\rho^*]} \, N\mathrm{Tr}_{k\times k} (R(u,z))^n \big|_{\rho=\rho^*}$

we want to make it look like $\int_{\text{brane}} \partial^{-1}\alpha$ ← KS field "sourced" by $N\mathrm{Tr}\, z^n$

we can define a surface $\Delta(u,x)$ s.t.

$$\mathrm{Tr}_{k\times k} R(u,x)^n = \int_{S_g^*} (b-ua)^n \, \delta_{\Delta(u,x)} \qquad (*)$$

$$R(u,x) = -g^{-1} \frac{\mu - u}{z-x} = -\frac{1}{z-x}\left(D(a) - uC(a) - zB(a) + zua \right) \quad \forall a$$

$\Delta(u,x)$: $\quad d - uc - xb + uxa = 0$

at k points where spectral curve S_g^* intersects surface $\Delta(u,z)$
$R(u,x)$ has an eigenvalue $(b-ua)$

$(*)$ will match B-model $\int_{\text{brane}} \partial^{-1}\alpha$ if we can identify

$$\alpha \Leftrightarrow \partial\left((b-ua)^n \, \delta_{\Delta(u,z)} \right)$$
$$= \partial\left((b-ua)^n \, \delta_{\frac{c}{a}=x} + (xa-c)^{-n} \, \delta_{\frac{b}{a}=u} \right)$$

Determinant modifications / brane excitations

modifications eg. $\det X \longrightarrow \frac{1}{N!}\,\varepsilon\varepsilon(\underbrace{X, X, \ldots}_{N-1}, Y^2)$

we can create mods of dets by acting with the global symm. alg.

global symmetry algebra of U_N:

$$\oint z^k A^{(n)}(z) = \oint z^k \,\mathrm{Tr}\, Z^{(i_1} Z^{i_2} \ldots Z^{i_n)} \quad , \quad 0 \leq k \leq n-2$$

$$\oint z^k B^{(n)}(z) \qquad 0 \leq k \leq n$$

$$\oint z^k C^{(n)}(z) \qquad 0 \leq k \leq n$$

$$\oint z^k D^{(n)}(z) \qquad 0 \leq k \leq n+2$$

focus on $A^{(n)}$ tower

we can organize the modes by their spin under $SL(2)_L$ and $SL(2)_R$

$$\mathcal{J}^{(n)}_{p,q} := \oint dz \; z^{p-1+\frac{n}{2}} \; N \; \mathrm{STr} \underbrace{XX\ldots}_{\frac{n}{2}+q} \underbrace{YY\ldots}_{\frac{n}{2}-q}(z)$$

(STr: symmetrized)

has spin p under $SL(2)_L$
q under $SL(2)_R$

includes $su(2)_R$ generators:

$$\mathcal{J}^{(2)}_{0,1} = \oint N \,\mathrm{Tr}\, XX \;, \quad \mathcal{J}^{(2)}_{0,0} = \oint N \,\mathrm{Tr}\, XY \;, \quad \mathcal{J}^{(2)}_{0,-1} = \oint N \,\mathrm{Tr}\, YY$$

acting with $\mathcal{J}^{(n)}_{p,q}$ on dets produces modifications

eg.

$$[\mathcal{J}^{(4)}_{-1,-4}, \det X(0)]$$

$$= \oint_{z=0} dz \; N \operatorname{Tr} Y^4(z) \det X(0)$$

$$\sim \varepsilon\varepsilon(X, X, X, \dots, Y^3) + \text{subleading}$$

Many modes $\mathcal{J}^{(n)}_{p,q}$ can create the same modifications so we do correlation functions:

$$\left< [\mathcal{J}^{(n')}_{p',q'}, \det Y(\infty)] \; [\mathcal{J}^{(n)}_{p,q}, \det X(0)] \right> \Big|_{\text{large } N}$$

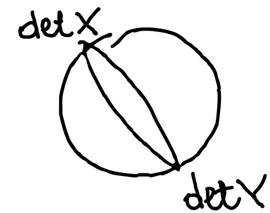
we got two types of det modifications:

* $\mathcal{J}^{(n)}_{p,p-1}$: $\det X(0) \longrightarrow n\, \mathcal{E}\mathcal{E}(X, \ldots, Y^{1-2p})$

* $\mathcal{J}^{(n)}_{p,p+1}$: $\det X(0) \longrightarrow n\, \mathcal{E}\mathcal{E}(X, \ldots, Y^{-2p-2}\partial X)$

$$+\, n\, \mathcal{E}\mathcal{E}(X, \ldots, \partial^2 Y^{-2p-3})$$

$\langle \det Y(\infty) \det X(0) \rangle$ has a single non-trivial saddle corresponding to brane:



$$g = \begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix} \subset SL(2,\mathbb{C})$$

holographic global symmetry algebra acts by holomorphic divergence-free vector fields on $SL(2,\mathbb{C})$

again we get two types of brane excitations:

* $J^{(n)}_{p,p-1}$: $\begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix} \longrightarrow \begin{pmatrix} a & \delta b \\ 0 & 1/a \end{pmatrix}$ $\qquad \delta b = \pm \varepsilon n a^{1-2p}$
* $J^{(n)}_{p,p+1}$: $\begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix} \longrightarrow \begin{pmatrix} a & 0 \\ \delta c & 1/a \end{pmatrix}$ $\qquad \delta c = \mp \varepsilon n a^{-1-2p}$

Future directions etc.

* calculate genus of S_g

* consider GG branes in presence of spacefilling branes

* find SUSY D3 branes in $AdS_5 \times S^5$ that correspond to our B-model D2 branes

* $\mathcal{O}(N^2)$: $(\det Z)^N$ $\longleftrightarrow$ backreacted geometries

* analyze which saddles actually contribute