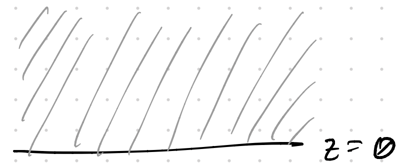
A PHYSICAL APPROACH TO TOPOLOGICAL HOLOGRAPHY

AdS/CFT

[Maldacena]

AdS: $$ds^2 = \frac{L^2}{z^2}\big(\overbrace{\eta_{\mu\nu}\, dx^\mu dx^\nu}^{d} + dz^2\big) \qquad d+1$$

$z > 0$

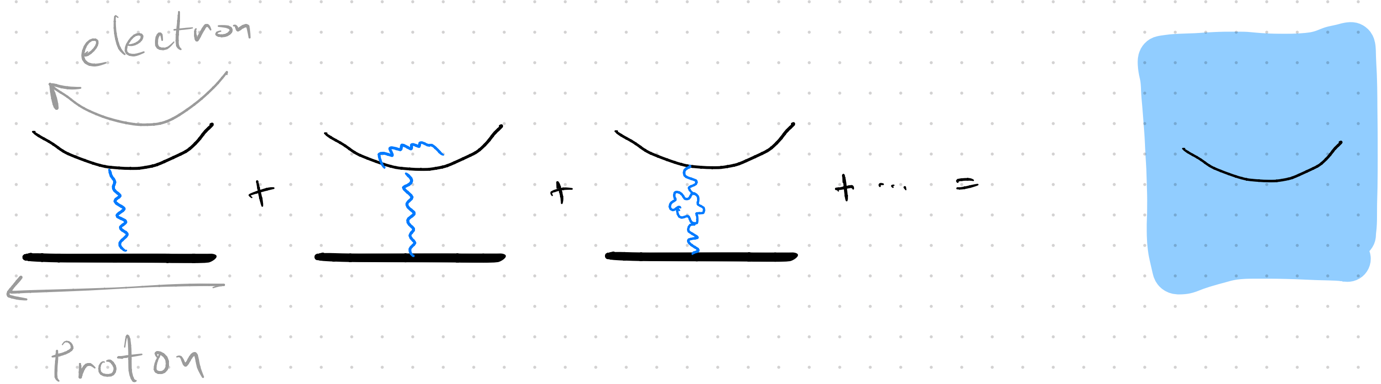


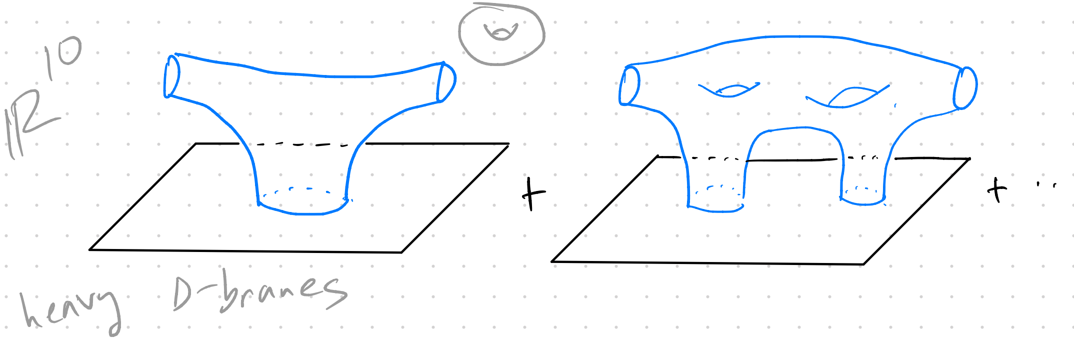
[Witten]

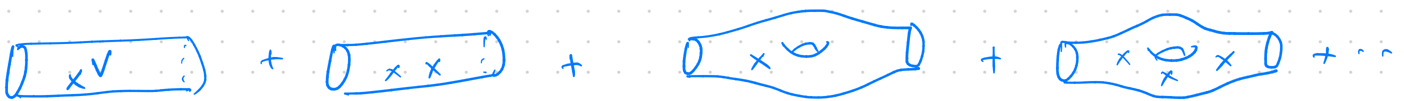
$$Z_{AdS}(\phi) = Z_{CFT}(\phi) = \left\langle e^{\int \phi \mathcal{O}} \right\rangle$$

boundary value

source





$=$ 

$$= \left\langle \int d^2\sigma \; V(\sigma) \right\rangle + \frac{1}{2!}\left\langle \int d^2\sigma \, d^2\sigma' \; V(\sigma)\, V(\sigma') \right\rangle + \cdots$$

$$= \left\langle e^{\int d^2\sigma \; V(\sigma)} \right\rangle$$

$\int d^2\sigma \; V(\sigma) \longrightarrow$ Background modification in worldsheet action

$\begin{array}{c}\text{Open strings} \\ \text{on D-branes}\end{array} + \begin{array}{c}\text{Closed strings} \\ \text{in } \mathbb{R}^{10}\end{array} + \text{Interactions}$

$=$ Closed strings in backreacted geometry

N D3 branes $=$ 4D $\mathcal{N}=4$ U(N) SYM

Backreacted geometry

$$ds^2 = \frac{1}{\sqrt{H(r)}} \eta_{\mu\nu} dx^\mu dx^\nu + \sqrt{H(r)}\, \delta_{ij} dy^i dy^j$$

$$H(r) = 1 + \frac{L^4}{r^4}$$

$\mu, \nu = 0, 1, 2, 3 \qquad i, j = 1, \ldots, 6$

$$L^4 = 4\pi g N \alpha'^2$$

$$r^2 = y_1^2 + \cdots + y_6^2$$

$$C_{(4)} = \left(1 - \frac{1}{H(r)}\right) dx^0 \wedge \cdots \wedge dx^3$$

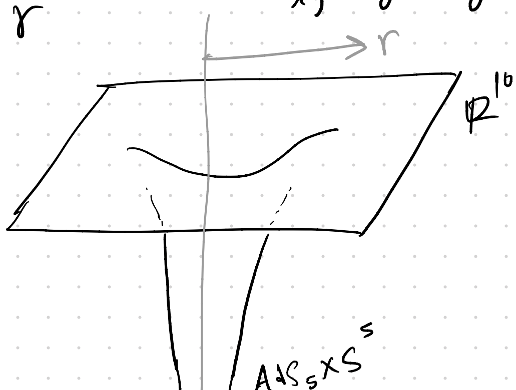
$$\int_{S^5} dC_{(4)} \propto N$$

$$ds^2 \xrightarrow{r\to\infty} \eta_{\mu\nu}\, dx^\mu dx^\nu + \delta_{ij}\, dy^i dy^j \qquad \mathbb{R}^{10}$$

$$ds^2 \xrightarrow{r\to 0} \frac{r^2}{L^2}\eta_{\mu\nu}\, dx^\mu dx^\nu + \frac{L^2}{r^2}\delta_{ij}\, dy^i dy^j$$

$$= \underbrace{\frac{L^2}{z^2}\left(\eta_{\mu\nu}\, dx^\mu dx^\nu + dz^2\right)}_{AdS_5} + \underbrace{L^2\, d\Omega_5^2}_{S^5}$$

$$z = \frac{L^2}{r} \qquad \delta_{ij}\, dy^i dy^j = dr^2 + r^2 d\Omega_5^2$$



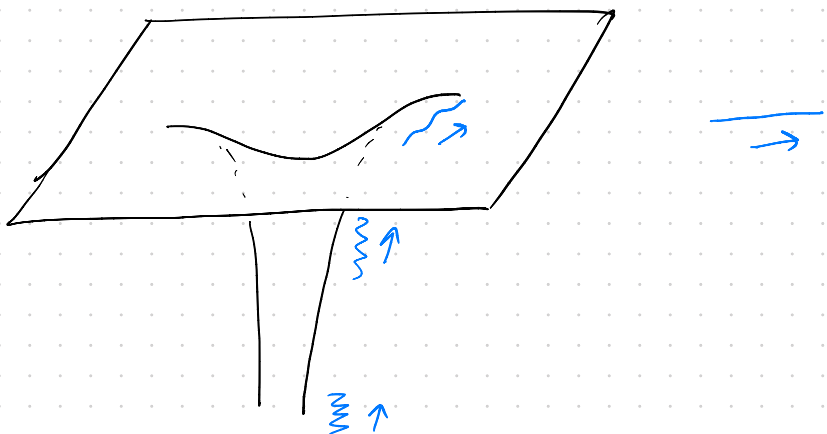
$r \to \infty \qquad ds^2 = dt^2 + \cdots$

time at ∞

$\int ds$

$r \to 0 \qquad ds^2 = \frac{r^2}{L^2} dt^2 + \cdots$

$$\frac{r}{L} \Delta\tau(\infty) = \Delta\tau(r) \Rightarrow \frac{r}{L} E(r) = E(\infty)$$



At low energy:

$$\mathcal{N}=4 \text{ SYM} + \text{massless } \mathbb{R}^{10} \text{ SUGRA}$$

$$= \text{Full } AdS_5 \times S^5 \text{ SUGRA} + \text{massless } \mathbb{R}^{10} \text{ SUGRA}$$

$$\Rightarrow \mathcal{N}=4 \text{ SYM} = \text{Full } AdS_5 \times S^5 \text{ SUGRA}$$

AdS boundary $\neq$ CFT world volume

Z_{AdS} (boundary values of fields)

$= Z_{CFT}$ (background source)

CFT can be coupled to AdS boundary

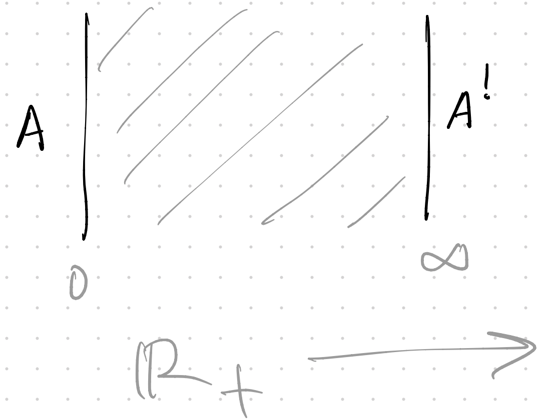
Local operators inside AdS $\xleftrightarrow[\text{dual}]{\text{Koszul}}$ Scatterings from boundary

$\parallel$

Local operators in CFT

Toy model: $\mathbb{R}^2 \times \mathbb{C} \setminus \mathbb{R} \rightsquigarrow \mathbb{R} \times \mathbb{R}_+ \times S^2$

Compactify S^2



	$\mathbb{R}^2_{+\epsilon}$	$\times\ \mathbb{R}^2_{-\epsilon}$	$\times\ \mathbb{C}$	$\times\ \mathbb{R}$	$\times\ \mathbb{R}$	$\times\ \mathbb{R}$	$\times\ \mathbb{R}$
N D3	$\mathbb{R}^2_{+\epsilon}\ \times$			$\mathbb{R}$	$\times\ \mathbb{R}$		
K D5		$\mathbb{R}^2_{-\epsilon}$	$\times\ \mathbb{C}$	$\times\ \mathbb{R}\ \times$		$\mathbb{R}$	

$N \to \infty$

$\mathcal{N}=4$ SYM with Wilson line

$=$ D5 branes wrapping $AdS_2 \times S^4 \subseteq AdS_5 \times S^5$

QM on intersection $= \int \mathrm{tr}_K\, \bar{\psi}\,(d + A_N)\,\psi + \mathrm{tr}_N\,(\psi A_K \bar{\psi})$

$\psi : \mathbb{C}^K \to \mathbb{C}^N$

$\bar{\psi} : \mathbb{C}^N \to \mathbb{C}^K$

$|\Omega\rangle$

$\psi, \bar{\psi}$

Ω-deformation

$\mathbb{R}^{10} \rightsquigarrow \mathbb{R}^2 \times \mathbb{R}_+ \times S^3$

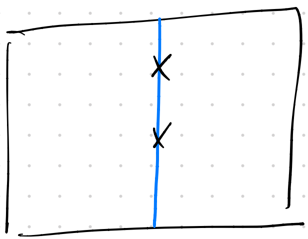
N D3 $\rightsquigarrow$ GL_N BF theory on $\mathbb{R}^2$

K D5 $\rightsquigarrow$ GL_K 4d CS on $\mathbb{C} \times \mathbb{R}^2$

$\mathbb{R} \times \mathbb{R}_+ \times S^2 \subseteq \mathbb{R}^2 \times \mathbb{R}_+ \times S^3$

Gauge

$$\frac{1}{\epsilon}\int_{\mathbb{R}^2} \mathrm{tr}_N (BF) + \frac{1}{\epsilon}\int_{\mathbb{R}} \mathrm{tr}_K \bar{\Psi}(d + A_N)\Psi + \sum_{n=0}^{\infty} \frac{1}{\epsilon}\int_{\mathbb{R}} \mathrm{tr}_K\left(\partial_z^n A_K \bar{\Psi} B^n \Psi\right)$$



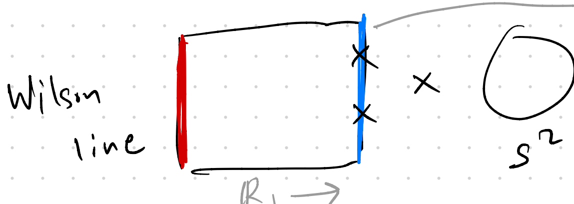
Operators on the

line: $O_j^i[n] := \frac{1}{\epsilon}\bar{\Psi}_j B^n \Psi^i$

They form some algebra $\mathcal{A}_{\text{gauge}}$

"Gravity"

$$\frac{1}{\epsilon}\int dz\ CS(A_K) + \text{a Wilson line}$$



"AdS boundary" with fixed value of A_K

$$\frac{\delta}{\delta \partial_z^m A^i_j} \qquad \frac{\delta}{\delta \partial_z^n A^k_l} \qquad Z_{CS}(A|_{\text{boundary}})$$

+Wils.

Generate some algebra $\mathcal{A}_{\text{grav}}$

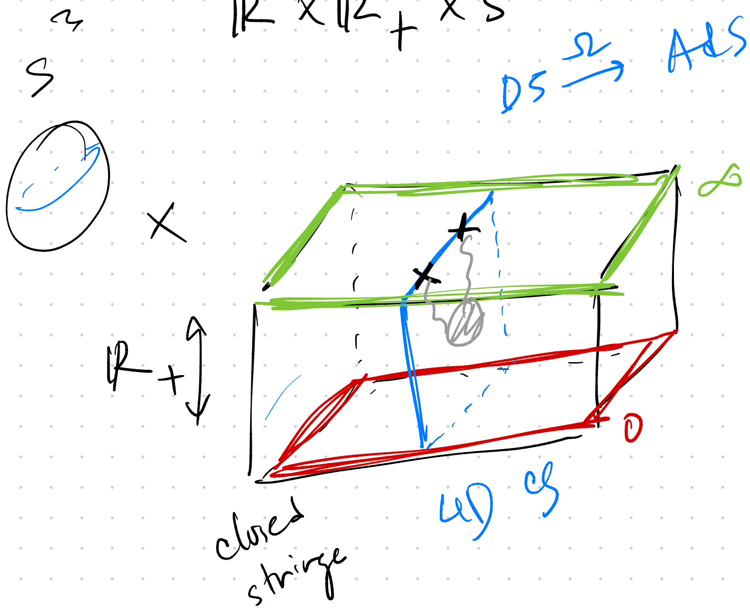
Holography : $\mathcal{A}_{\text{gauge}} = \mathcal{A}_{\text{grav}} = Y(gl_K)$

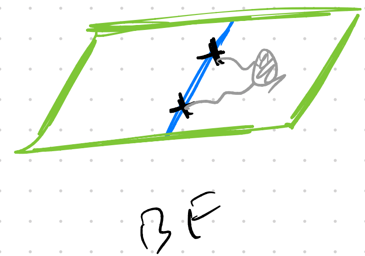
$$AdS_3 \times S^3 \longleftrightarrow CFT_2$$

$$\mathbb{R}^2 \times \mathbb{R}_+ \times S^3$$

$$D5 \xrightarrow{\Omega} AdS_2 \times S^2$$

$$\underline{\mathbb{R} \times \mathbb{R}_+ \times S^2}$$





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